Practice Test Answer and Alignment Document

Mathematics – Algebra 1

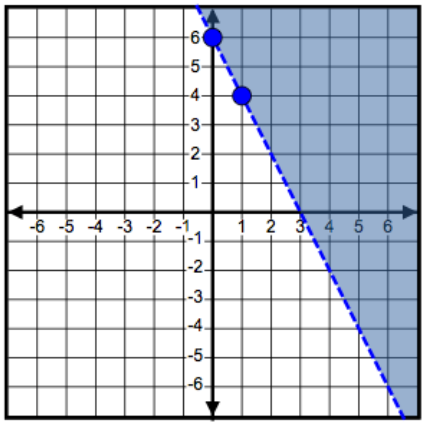
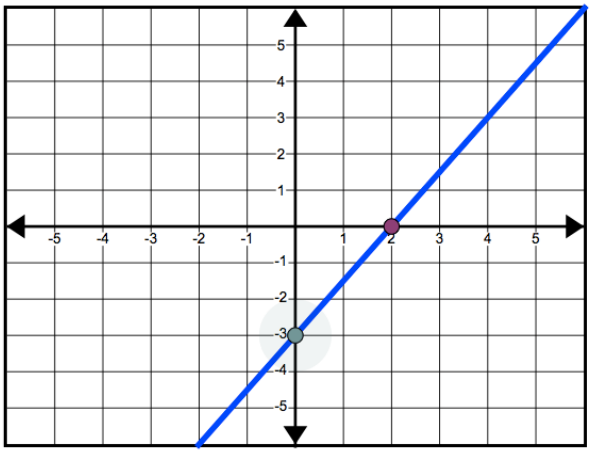
Online

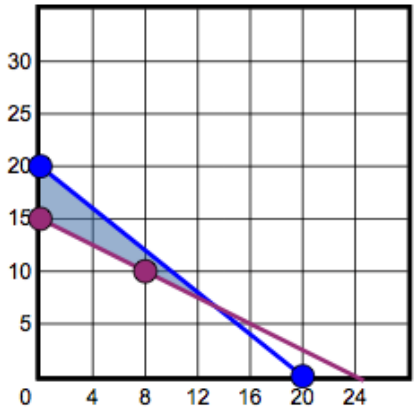
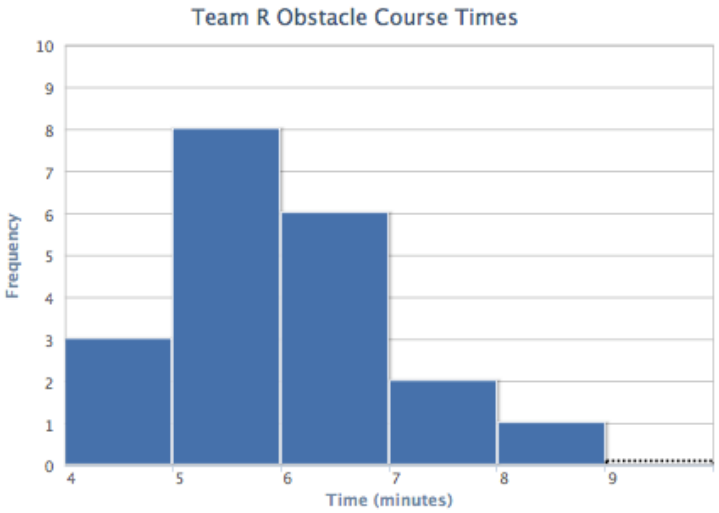
The following pages include the answer key for all machine-scored items, followed by the rubrics for the hand-scored items.

- The rubrics show sample student responses. Other valid methods for solving the problem can earn full credit unless a specific method is required by the item.
- In items where the scores are awarded for full and partial credit, the definition of partial credit will be confirmed during range-finding (reviewing sets of real student work).
- If students make a computation error, they can still earn points for reasoning or modeling.

Unit 1

Item Number	Answer Key	Evidence Statement Key / Content Scope	Integrated Course Alignment
1.	−4 2 6 in any order.	A-APR.3-1	3
2.	Part A: Value | $a + b$ | $a - b$ | c^2 Always Rational | ● | ● | ○ Never Rational | ○ | ○ | ○ Sometimes Rational | ○ | ○ | ●	N-RN.B-1	2

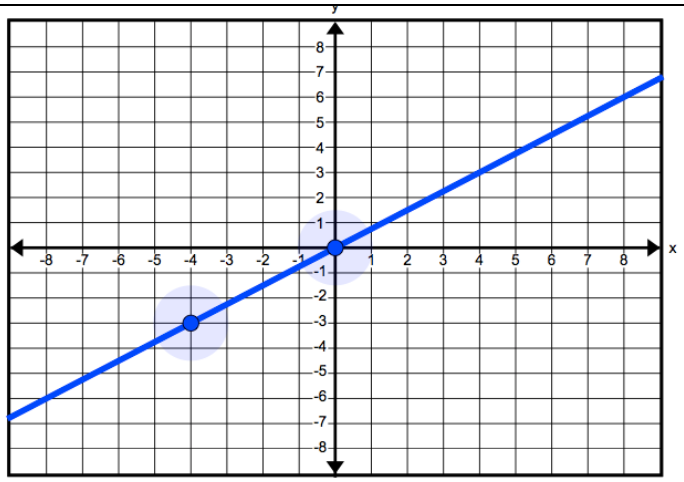
	Part B: B		
3.		A-REI.12	1
4.	B	A-SSE.3a	2
5.	C	A-APR.1-1	2
6.	Part A: $x + y = 16$ $2x + 3y = 39$ Part B: 7	A-REI.6-1	1
7.	-3	F-BF.3-1	2
8.	$f(2)$ [is greater than] $g(2)$ and $f(-2)$ [is less than] $g(-2)$.	F-IF.9-1	none
9.		F-IF.7a-1	1
10.	If $C(4) = 398$, then [4] pairs of sunglasses cost $ [398.00]	F-IF.A.Int.1	1
11.	Part A:	A-CED.3-1	1

	 Part B: A, B, C Part C: 11 Part D: 13		
12.	Part A:  Part B: A, C	S-ID.Int.2	none
13.	Part A: see rubric Part B: see rubric Part C: see rubric	HS.D.1-1/ 8.EE.C.07.b	1
14.	See rubric	HS.D.3-1/ A-CED.A.01 7.RP.A.03	1

Unit 2

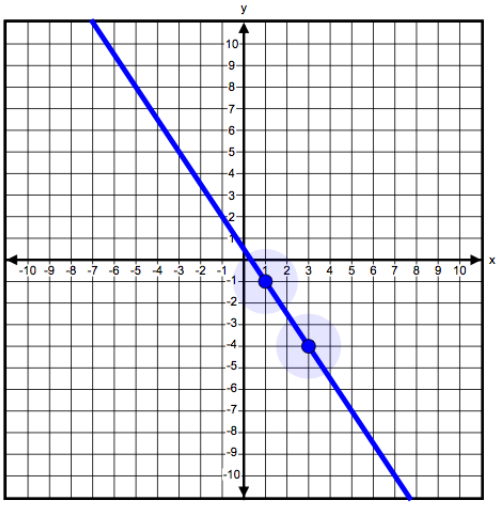
Item Number	Answer Key	Evidence Statement Key/Content Scope	Integrated Course Alignment

1.	A, C, D	S-ID.5	1
2.	3	A-REI.11-1b	1
3.	See rubric	HS.C.6.1/ A-REI.D.10 A-REI.D.11	1
4.	B	A-SSE.1-1	1
5.	B	A-CED.4-1	1
6.	Part A: $x < 0$ Increasing $0 < x < 2$ Increasing $2 < x < 4$ Decreasing $x > 4$ Decreasing Part B: $x < 0$ $f(x) < 0$ $0 < x < 2$ $f(x) > 0$ $2 < x < 4$ $f(x) > 0$ $x > 4$ $f(x) < 0$	F-IF.4-1	none
7.	8, -8	A-REI.4b-1	2
8.	Part A: see rubric Part B: see rubric	HS.D.2-5/ A-CED.A.01	1
9.	D	F-IF.5-1	1
10.	Part A: $R(x) =$ (1,000−20x) × (10+x) Part B: B, C, D, E	HS-Int.1	none
11.	C, E	A-REI.10	1
12.	The average rate of change in the number of bacteria for the first 5 minutes of the experiment is 24.8 bacteria per minute .	F-IF.6-1a	None
13.	Part A:	HS.C.18.1/ 8.EE.B.05	1

	 Part B: see rubric Part C: see rubric		
14.	B	A-CED.4-2	2
15.	A, D	F-IF.1	1

Unit 3

Item Number	Answer Key	Evidence Statement Key / Content Scope	Integrated Course Alignment
1.	$N(t) = 150(3)^t$	F-LE.2-1	1
2.	Part A: When the equation is graphed in a coordinate plane, the *x*-intercepts are (0,0) and (18,0) , which represen a lower bound and an upper bound for the possible values for the width of the pen. Part B: 9	F-IF.8a	2
3.	A, C, E	A-REI.4b-2	2
4.	Part A: $f(x) = 2(x + 1.5)^2 + -12.5$ Part B:	A-SSE.2-4	none

	$f(x) = 2(x +$ [4] $)(x +$ [-1] $)$		
5.	See rubric	HS.C.12.1/ F-IF.C.08.a	2
6.	C	A-SSE.1-2	2
7.	Part A: A Part B: C	F-Int.1-1	none
8.		A-REI.10	1
9.	Part A: see rubric Part B: see rubric	HS.D.2-9/ F-BF.A.01.a	2
10.	See rubric	HS.C.16.2/ A-REI.B.04.a A-REI.B.04.b	2
11.	Part A: 51600 Part B: $36000(1.04)^t$ Part C: 53289 Part D: C	HS-Int.3-1	1
12.	A	F-IF.6-6b	none

Rubrics start on the next page.

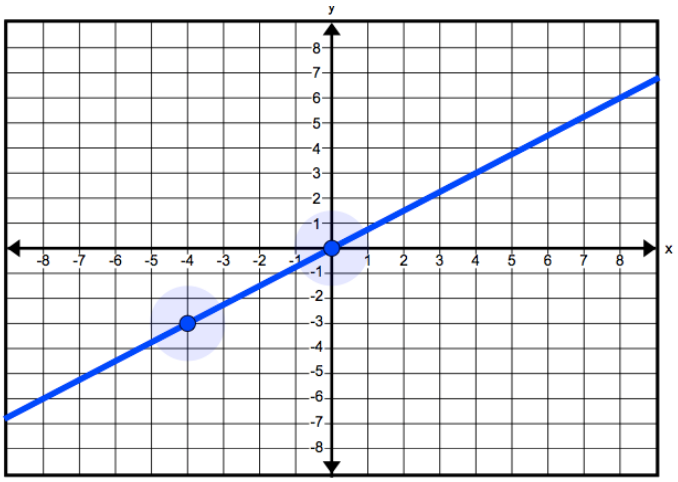
Unit 1 #13 Part A	
Score	**Description**
2	Student response includes each of the following 2 elements: • Correct equation • Valid justification of how the equation was determined Sample Student Response: Let m be the number of cookies that Matt made. Then the number of cookies that Phil made would be $1.25m$. Let A represent the total amount of money earned. $A = 0.25(0.80)(m + 1.25m)$ The total number of cookies made is the sum of the number Matt made and the number Phil made. Only 80% of the cookies sold, so the total number needs to be multiplied by 0.8. Each cookie sold for \$0.25, so the total amount earned would be 0.25 times the 80% that were sold.
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.
Unit 1 #13 Part B	
2	Student response includes each of the following 2 elements: • Determination that Matt made 160 cookies and Phil made 200 cookies • Valid work shown Sample Student Response: $72 = 0.25(0.80)(m + 1.25m)$ $72 = (0.20)(2.25m)$ $72 = 0.45m$ $160 = m$ $1.25m = 1.25(160) = 200$ Matt made 160 cookies and Phil made 200 cookies. Note: Student may earn the points in Part B by correctly using an incorrect equation from Part A.
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.

Unit 1 #13 Part C	
2	Student response includes the following element: • Full justification for raising the price Sample Student Response: If they raise the price to \$0.50 and only sell 70% of the cookies, the equation will be $A = 0.5(0.70)(160 + 200)$. In this case they will make \$126, which is over \$50 more than they made this year. They should raise the price of the cookies. Note: The student may give a valid reason for not raising the price based on risk. This should still earn credit. Also, the student may earn the points in Part C by correctly using an incorrect equation from Part A or B.
1	Student response includes partial justification for raising the price.
0	Student response is incorrect or irrelevant.

Unit 1 #14 Rubric	
Score	**Description**
3	Student response includes each of the following 3 elements: • Speed of riverboat for current tour • Speed or time of riverboat for shorter tour • Decision with justification Note: There is no reason given in the task on why the riverboat takes longer going upstream than downstream. One reason could be the current of the river. Another could be that there is more sightseeing happening upstream and the boat tends to slow down. Whatever the case, the decision should be made whether to treat the upstream/downstream trip separately or as one. Either is fine, but the response should be consistent in all parts. Sample Student Response: Traveling upstream, the riverboat goes 25 miles in 2 hours giving it an average speed of 12.5 mph. Downstream, it travels the same distance in less time, going a distance of 25 miles in 1.25 hours. This gives an average speed of 20 mph. Using the upstream speed of 12.5 mph, a 20-mile trip would take 1.6 hours. Using the downstream speed of 20 mph, the trip back would take 1 hour. The entire trip would take 2.6 hours. A 20-mile tour upstream and back again would take about 6 minutes longer than the desired 2.5 hours (2 hours 30 minutes). So technically the answer is no, such a trip is not possible using the current speeds. But possibly increasing the speed only slightly could allow for the trip to fit into the desired time constraint. Note: Students could instead use a single speed approach. For example, the total trip of 50 miles takes 3.25 hours giving an average speed of 15.4 mph. The shorter tour of 40 miles traveling 15.4 mph would take about 2.6 hours.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

Unit 2 #3	
Score	**Description**
3	Student response includes each of the following 3 elements: • Correct justification of the number of points on the graph for $c < 0$ • Correct justification of the number of points on the graph for $c = 0$ • Correct justification of the number of points on the graph for $c > 0$ Sample Student Response: • $\lvert x \rvert$ and $\lvert y \rvert$ are each nonnegative for all real numbers x and y. So, the sum must be nonnegative for all real numbers. Therefore, the sum cannot equal a negative number. There are no solutions and no points on the graph $c > 0$ • If $c = 0$, there is only one solution, $(0, 0)$. The graph consists of only one point. • If $c > 0$, there are infinitely many solutions, which means that there are infinitely many points on the graph.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

Unit 2 #8 Part A	
Score	**Description**
1	Student response includes the following element: • Correct model Sample Student Response: $x + (x - 50) + (x - 100) + (x - 150) + (x - 200) = P$ Where x is the amount of money for the first place prize and P is the total amount of prize money.
0	Student response is incorrect or irrelevant.
Unit 2 #8 Part B	
2	Student response includes each of the following 2 elements: • Correct amounts for each of the five prizes • Valid work shown Sample Student Response: $x + (x - 50) + (x - 100) + (x - 150) + (x - 200) = 1000$ $5x - 500 = 1000$ $5x = 1500$ $x = 300$ Fifth place is \$100, fourth place is \$150, third place is \$200, second place is \$250, and first place is \$300.
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.

Unit 2 #13 Part A	
Score	**Description**
1	
Unit 2 #13 Part B	
2	Student response includes each of the following 2 elements: • Explanation of why the coordinate values have the same ratio • Explanation of the exception regarding the y-intercept Sample Student Response: The graph passes through the origin, so if (x, y) is a point on the line, then the slope can be represented by $\frac{y-0}{x-0}$ which is the same as the ratio of the coordinate values. Because the slope is constant, the ratio is the same for all points on the line, with the exception of the y-intercept which is $(0, 0)$. The y-intercept (the origin) does not work because 0 divided by 0 is undefined.
1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.

Unit 2 #13 Part C	
1	Student response includes the following element: • Explanation why the line does not have the same property as in Part B Sample Student Response: The equation $y = 3x - 2$ has a y-intercept of -2, so the line will not pass through the origin. As a result, the line will not have the same property as in Part B.
0	Student response is incorrect or irrelevant.

Unit 3 #5	
Score	**Description**
4	Student response includes each of the following 4 elements: • Algebraic reasoning about the point $(2 + d, y)$ • Algebraic reasoning about the point $(2 - d, y)$ • Identification of the line of symmetry, $x = 2$ • Justification of the line $x = 2$ as the line of symmetry of $f(x)$ Sample Student Response: If $(2 + d, y)$ is on the graph of f, then: $y = f(2 + d) = (2 + d)(2 + d - 4)$ $= (2 + d)(d - 2)$ $= d^2 - 4$ Therefore, $d^2 - 4$ equals y. If $(2 - d, y)$ is on the graph of f, then: $y = f(2 - d) = (2 - d)(2 - d - 4)$ $= (2 - d)(-d - 2)$ $= d^2 - 4$ $= y$ Therefore, $y = y$, so if the point $(2 + d, y)$ is on the graph of f, then so is $(2 - d, y)$. The line $x = 2$ is a line of symmetry for the graph of f. I know this because x-values that are the same distance (absolute value) d from 2 yield equal y-values in the function. Notes: • Correct simplification is not necessary to earn the first point. • To earn the second point, the two expressions must match and have no mistakes. • The student may appeal to a formula (such as $x = -\frac{b}{2a}$) for the line of symmetry. • Any justification that addresses point pairs on either side of the line is accepted.
3	Student response includes 3 of the 4 elements.
2	Student response includes 2 of the 4 elements.
1	Student response includes 1 of the 4 elements.
0	Student response is incorrect or irrelevant.

Unit 3 #9 Part A	
Score	**Description**
3	Student response includes each of the following 3 elements: • Correct model • Valid work shown • Valid explanation of d with relation to 450. Sample Student Response: For 20 minutes of shower time, the family can save $(5 - 3)(20) = 60$ gallons each day. At \$0.002 per gallon, this is a savings of \$0.12 per day. Let S represent the cost savings, in dollars, and let d represent the time in days: $S = -54 + 0.12d$ The number of days at which the savings become zero can be found by solving this equation: $-54 + 0.12d = 0$ $0.12d = 54$ $d = 450$ For values of d less than 450, the savings due to reduced water consumption have not yet exceeded the cost of the low-flow showerhead. For values of d greater than 450, the savings due to reduced water consumption have exceeded the cost of the low-flow showerhead. Therefore, the cost savings will be greater than zero after 450 days.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

Unit 3 #9 Part B	
3	Student response includes each of the following 3 elements: • Correct model • Valid work shown • Correct computation and interpretation of 81 Sample Student Response: In the first year, the savings in water costs are $(365)(\$0.12) = \$43.80$. The low-flow showerhead costs \$54, and so there is still $\$54 - \$43.80 = \$10.20$ to recover. After the first year, the cost savings will be $(12)(1.05) = 12.6$ cents, or \$0.126 per day. So if S represents the savings and d_2 represents the number of days in the second year, then the new model is: $S = -10.2 + 0.126d_2$ The number of days at which the savings become zero can be found by solving this equation: $-10.2 + 0.126d_2 = 0$ $0.126d_2 = 10.2$ $d_2 \approx 81$ The family will start saving money 81 days into the second year. Note: The student will earn the point if he or she correctly interprets his or her reasonable incorrect model.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

Unit 3 #10	
Score	**Description**
3	Student response includes each of the following 3 elements: • Correct process for deriving the solution • Correctly states the conditions under which x is a real number when $a = 2$ and $b = 5$, which is that c must be greater than or equal to negative 5 • Correct reasoning shown to support the conditions under which x is a real number when $a = 2$ and $b = 5$ Sample Student Response: $a(x-3)^2 - b = c$ $a(x-3)^2 = b + c$ $(x-3)^2 = \frac{b+c}{a}$ $x - 3 = \pm\sqrt{\frac{b+c}{a}}$ $x = 3 \pm \sqrt{\frac{b+c}{a}}$ If $a = 2$ and $b = 5$, then $x = 3 \pm \sqrt{\frac{5+c}{2}}$. For x to be a real number, $\frac{5+c}{2}$ must be greater than or equal to zero. Therefore, c must be greater than or equal to -5.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.