Practice Test Answer and Alignment Document

Mathematics – Geometry

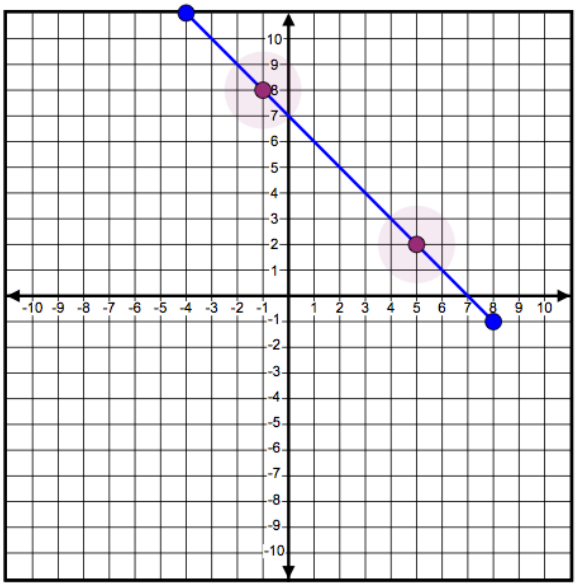
Online

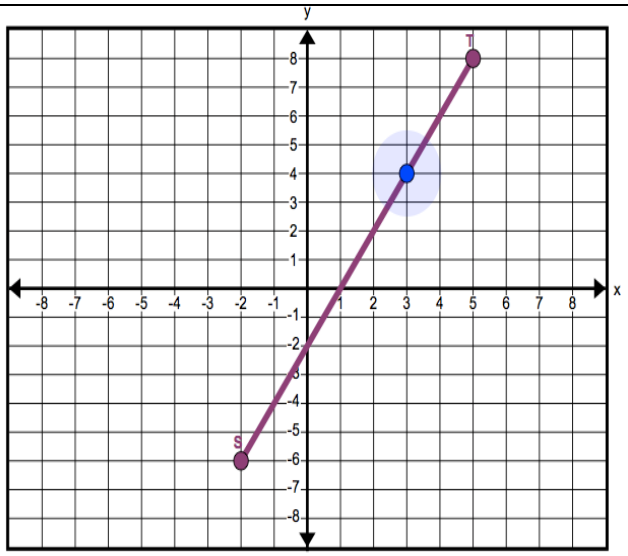
The following pages include the answer key for all machine-scored items, followed by the rubrics for the hand-scored items.

- The rubrics show sample student responses. Other valid methods for solving the problem can earn full credit unless a specific method is required by the item.
- In items where the scores are awarded for full and partial credit, the definition of partial credit will be confirmed during range-finding (reviewing sets of real student work).
- If students make a computation error, they can still earn points for reasoning or modeling.

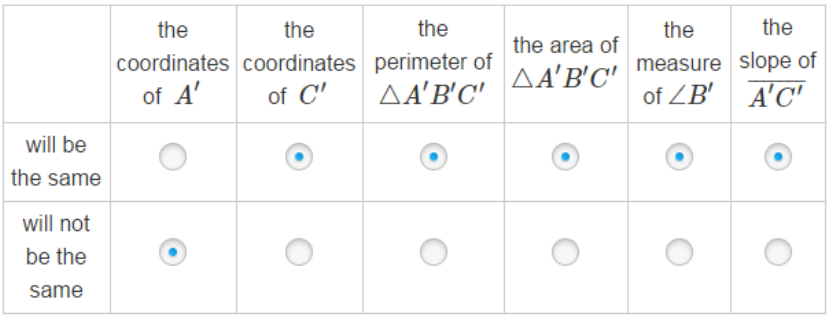
Unit 1

Item Number	Answer Key	Evidence Statement Key/Content Scope	Integrated Course Alignment
1.	C	G-SRT.1a	2
2.	Part A: -1 Part B: 44	G-GPE.1-2	3

3.		G-GPE.6	3
4.	15	G-SRT.5	2
5.	Part A: Statement: $\angle CBD \cong \angle BFE$ Reason: Given Statement: $\angle CBD \cong \angle ABF$ Reason: Vertical angles are congruent Statement: $\angle ABF \cong \angle BFE$ Reason: Transitive property of congru Part B: Statement: $m\angle CBD = m\angle BFE$ Reason: Given Statement: $m\angle CBD + m\angle DBF = 180°$ Reason: Angles that form a linear pair Statement: $m\angle BFE + m\angle DBF = 180°$ Reason: Substitution property of equa	G-CO.C	1
6.	A	G-SRT.6	2

<table>
<tr><td>7.</td><td> 
</td><td>G-GPE.6</td><td>3</td></tr>
<tr><td>8.</td><td>
<table>
<tr><th>Subtended Central Angle</th><th>Arc Length</th></tr>
<tr><td>$\angle AFB$</td><td>2π</td></tr>
<tr><td>$\angle BFC$</td><td>$\frac{3\pi}{4}$</td></tr>
<tr><td>$\angle CFD$</td><td>$\frac{\pi}{2}$</td></tr>
<tr><td>$\angle AFE$</td><td>π</td></tr>
</table>
</td><td>G-C.B.Int.1</td><td>3</td></tr>
<tr><td>9.</td><td>Triangle KLM is similar to $\Delta K'L'M'$, which we can determine by a dilation of scale factor 1.5 centered at the origin.</td><td>G-SRT.2</td><td>2</td></tr>
<tr><td>10.</td><td>E</td><td>G-SRT.7-2</td><td>2</td></tr>
<tr><td>11.</td><td>Part A: B
Part B: A</td><td>G-CO.D</td><td>3</td></tr>
<tr><td>12.</td><td>Part A: 243.2
Part B: 1740
Part C: 42.8
Part D: A, B, C</td><td>G-Int.1</td><td>None</td></tr>
<tr><td>13.</td><td>See rubric</td><td>HS.D.3-2</td><td>2</td></tr>
<tr><td>14.</td><td>Part A: see rubric
Part B: see rubric</td><td>HS.D.1-2</td><td>2</td></tr>
</table>

Unit 2

Item Number	Answer Key	Evidence Statement Key/Content Scope	Integrated Course Alignment
1.	The triangles are similar because $\triangle DEF$ is the image of $\triangle ABC$ under a dilation with center (-2, -1) and scale factor 2.	G-SRT.2	2
2.	B	G-CO.3	1
3.	Part A: 3.4 Part B: C	G-SRT.8	2
4.	C	G-GMD.4	3
5.	See rubric	HS.D.2-2	2
6.		G-CO.6	1
7.	D	G-SRT.5	2
8.	Part A: see rubric Part B: see rubric	HS.C.13.2	3
9.	Line $A'C'$ will be parallel to $\overleftrightarrow{AC}$, and line $\overleftrightarrow{P'Q'}$ will be the same line as $\overleftrightarrow{PQ}$.	G-SRT.1a	2
10.	Part A: B, D Part B: 3	G-CO.5	1
11.	To find the center of the circle and the length of the radius, the equation can be rewritten as $(x-4)^2 + y^2 = 25$.	G-GPE.1-2	3

12.	The scale factor is [0.5], and the center of dilation is at ([0] , [0]).	G-SRT.1b	2
13.	When the probe reaches the ocean floor, the probe will be approximately [3,600] meters below the ocean surface. When the probe reaches the ocean floor, the horizontal distance of the probe behind the team's ship on the ocean surface will be approximately [1,500] meters.	G-SRT.8	2
14.	Part A: 36 Part B: The length of $\overline{CD}$ is [greater than 10] because [m∠CPD > 60°].	G-CO.D	3
15.	Part A: see rubric Part B: see rubric	HS.C.18.2	None
16.	B, F	G-CO.1	1

Unit 3

Item Number	Answer Key	Evidence Statement Key / Content Scope	Integrated Course Alignment
1.	B	G-GMD.3	2
2.	C	G-SRT.7-2	2
3.	Part A : B Part B: The construction creates congruent triangles. Triangle ABD and $\triangle ACD$ are congruent because of the [side, side, side] theorem. It follows that $\overrightarrow{AD}$ must be the angle bisector of $\angle BAC$ because [∠BAD ≅ ∠CAD].	G-CO.D	3

4.	46	G-C.2	3
5.	See rubric	HS.C.14.5	2
6.	A, D	G-CO.1	1
7.	Part A: 2000 Part B: 1732 Part C: B Part D: C	G-SRT.8	None
8.	A	G-SRT.1b	2
9.	Part A: see rubric Part B: see rubric Part C: see rubric	HS.D.2-11	2
10.	B, F	G-GMD.1	2
11.	Part A: see rubric Part B: see rubric	HS.C.14.3	3
12.	C	G-CO.6	1
13.	Part A : 1326 Part B : B	G-Int.1	3

Rubrics start on the next page.

Unit 1 #13 Rubric	
Score	**Description**
3	Student response includes each of the following 4 elements: • Determines the value of x to be about 16 feet • Creates a valid equation • States assumptions needed to use the equation • Valid work shown Sample Student Response: There is an assumption that the two triangles (the one formed by the billboard and the one formed by the post) are similar and very close to right triangles. $\frac{x}{26} = \frac{4}{\sqrt{4^2 + 5^2}}$ $x = 26\left(\frac{4}{\sqrt{41}}\right)$ $x \approx 16.24$ Note: An alternate equation is to let y be the angle formed by the support and the ground. Then $y = \cos^{-1}\left(\frac{4}{\sqrt{41}}\right)$, and $x = 26\cos y$. Note: Any intermediate rounding should also result in an answer of about 16. Note about assumptions: There is no indication in the description that the triangles are right triangles, although the figure certainly suggests it. Possible assumptions should address this if the Pythagorean theorem or the cosine function is used, such as: • There is an assumption that the billboard and the post are both perpendicular to the ground and that the ground is level; or • There is an assumption that the billboard and the post are parallel and very nearly perpendicular to the ground, and that the ground is level.
2	Student response includes 3 of the above elements.
1	Student response includes 1 or 2 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 1 #14 Part A Rubric	
Score	**Description**
3	Student response includes each of the following 3 elements: • Correct identification of the shape of the exposed surface as a rectangle • Correct area of the rectangle • Valid work shown Sample Student Response: The shape of the exposed surface is a rectangle. The width of this rectangle is the length of one edge of the cube, which is 10 inches. The length of the rectangle, *d,* is the length of the diagonal of a square face of the cube. To find this length, apply the Pythagorean Theorem. $c^2 = a^2 + b^2$ $d^2 = 10^2 + 10^2$ $d^2 = 200$ $d = \sqrt{200} \approx 14.1$ (inches) The length of the diagonal of a face of the cube is approximately 14.1 inches. The area of the rectangle (exposed surface) is $10\sqrt{200} \approx 141$ square inches.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

Unit 1 #14 Part B Rubric	
Score	**Description**
3	Student response includes each of the following 3 elements: • Correct conclusion that the volume of clay in the chunk is 500 cubic inches • Correct conclusion that the volume of each clay sphere is $\frac{32}{3}\pi$ cubic inches, or approximately 33.5 cubic inches • Correct conclusion that Daniel can make 14 clay spheres, with calculations to support that conclusion Sample Student Response: The volume of each congruent chunk is half the volume of the entire block. The volume of the cube is $(10 \text{ inches})^3$, or 1,000 cubic inches. So the volume of each congruent chunk of clay is 500 cubic inches. Each sphere will have a diameter of 4 inches and a radius of 2 inches. The volume of each clay sphere will be $\frac{4}{3}\pi (2 \text{ inches})^3$, or $\frac{32}{3}\pi$ cubic inches. This is approximately 33.5 cubic inches. To find the number of spheres that Daniel can make from the chunk of clay, divide the volume of the full chunk of clay, by the volume of one sphere: $500 \div 33.5 \approx 14.9$. The result of 14.9 means that there is enough clay in the chunk to make 14 clay spheres because there is not enough clay to make 15 complete spheres.
2	Student response includes 2 of the above elements.
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 2 #5 Rubric	
Score	**Description**
3	Student response includes each of the following 3 elements: • Valid values for h_1 and h_2 • Valid approach for determining h_1 and h_2 • Verification that the design will store at least 2 dump-truck loads of fertilizer Sample Student Response: Assuming the dump trucks are rectangular prisms, each dump truck stores 288 cubic feet of fertilizer ($4 \times 6 \times 12 = 288$). Two dump trucks will store 576 cubic feet of fertilizer. The volume of the storage building needs to be at least 576 cubic feet. The volume of the storage building equals the volume of the cylinder plus the volume of the cone. I used the maximum diameter of 8 feet. $\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$ $\pi 4^2 h_1 + \frac{1}{3}\pi 4^2 h_2$ I used the maximum total height of 14 feet. Since the volume of a cone involves dividing by 3, I made the height of the cone much smaller than the height of the cylinder. $\pi 4^2 h_1 + \frac{1}{3}\pi 4^2 h_2$ $\pi 4^2(11) + \frac{1}{3}\pi 4^2(3) \approx 603.16$ Using $h_1 = 11$ feet and $h_2 = 3$ feet, the storage building will have a volume greater than 576 cubic feet. Note: Any two heights that have a sum of 14 and create a volume greater than 576 are acceptable.
2	Student response includes 2 of the above elements.
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 2 #8 Part A Rubric	
Score	**Description**
1	Student response includes the following element: • Correct coordinates of point Q in terms of a, b, and c. Sample Student Response: $(2a + 2b, 2c)$ or equivalent Note: Students are not required to show work, but will not be penalized for showing work.
0	Student response is incorrect or irrelevant.

Unit 2 #8 Part B Rubric	
Score	**Description**
2	Student response includes each of the following 2 elements: • Student states that the midpoint of $\overline{SQ}$ must be the same as the midpoint of $\overline{PR}$ • Provides evidence using appropriate mathematical strategies, reasoning, and/or approaches that verifies $\overline{SQ}$ and $\overline{PR}$ bisect each other Sample Student Response: $\overline{SQ}$ bisects $\overline{PR}$ and $\overline{PR}$ bisects $\overline{SQ}$. I know this because the midpoint of $\overline{SQ}$ has the same coordinates as the midpoint of $\overline{PR}$, as shown. midpoint of $\overline{SQ} = \left(\frac{0+2a+2b}{2}, \frac{0+2c}{2}\right)$ $= \left(\frac{2a+2b}{2}, \frac{2c}{2}\right)$ $= (a + b, c)$ midpoint of $\overline{PR} = \left(\frac{2a+2b}{2}, \frac{0+2c}{2}\right)$ $= \left(\frac{2a+2b}{2}, \frac{2c}{2}\right)$ $= (a + b, c)$ Since the point of intersection of $\overline{SQ}$ and $\overline{PR}$ is the midpoint of both segments, $\overline{SQ}$ and $\overline{PR}$ bisect each other.
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 2 #15 Part A Rubric	
Score	**Description**
3	Student response includes each of the following 3 elements: • Determination that $BE = 16$ and $DE = 16$ • Correct algebraic reasoning • Recognition of an extraneous solution Sample Student Response: Because the figure is a parallelogram, the diagonals bisect each other. $x^2 - 48 = 2x$ $x^2 - 2x - 48 = 0$ $(x - 8)(x + 6) = 0$ $x = 8$ or $x = -6$ $x = -6$ is not possible because $2x$ would equal -12 and length is not negative. So $x = 8$, $BE = 8^2 - 48 = 16$, and $DE = 2(8) = 16$ Notes: • If the student makes an error in writing the equation and gets two answers that work, the student must show both solutions to earn the second point. • If the student makes a computation error and finds that the diagonals are not congruent, the third reasoning point can be earned if the student concludes that the figure is not a rectangle.
2	Student response includes 2 of the above elements.
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 2 #15 Part B Rubric	
Score	**Description**
1	Student response includes the following element: • Explanation of why the parallelogram is a rectangle or that diagonals are congruent. Sample Student Response: *ABCD* is a rectangle. Because the lengths *AE*, *BE*, *CE*, and *DE* are all equal to 16, the diagonals are congruent and parallelogram *ABCD* is a rectangle.
0	Student response is incorrect or irrelevant.

Unit 3 #5 Rubric	
Score	**Description**
3	Student has complete valid proof with valid chains of reasoning. Student Sample Response: Given: $\overline{A'B'}$ is the image of $\overline{AB}$ after a dilation centered at point C and with scale factor k, $k > 0$. Prove: $A'B' = k \cdot AB$. 1. $\overline{A'B'}$ is the image of $\overline{AB}$ after a dilation centered at point C and with scale factor k, $k > 0$. [given] 2. $CA' = k \cdot CA$; $CB' = k \cdot CB$ [definition of dilation] 3. $\frac{CA'}{CA} = k$; $\frac{CB'}{CB} = k$; [Division Property of Equality] 4. $\frac{CA'}{CA} = \frac{CB'}{CB}$ [Transitive Property of Equality] 5. $\angle C \cong \angle C$ [Reflexive Property of Congruence] 6. $\triangle ACB \sim \triangle A'CB'$ [SAS Triangle Similarity] 7. $\frac{A'B'}{AB} = \frac{CA'}{CA} = \frac{CB'}{CB}$ [definition of similar polygons] 8. $\frac{A'B'}{AB} = k$ [Substitution Property of Equality] 9. $A'B' = k \cdot AB$ [Multiplication Property of Equality]
2	Student response contains a valid chain of reasoning but may omit some information. The student must include the SAS Triangle Similarity property correctly.
1	Student response includes valid justified steps, but is an incomplete proof.
0	Student response is incorrect or irrelevant.

Unit 3 #9 Part A Rubric	
Score	**Description**
2	Student response includes each of the following 2 elements: • The surface area of the roof is approximately 396 square foot • Valid work shown Sample Student Response: The angle consists of two parts – the right angle created by the side of the shed, and the top of the rectangular side and the acute angle used to form a right triangle. For a 125° angle, the right triangle of the roof line has an angle of 35° and an adjacent side of 9. Each side of the roof is a rectangle with one side measuring 18 feet and the other side measuring $\frac{9}{\cos 35}$ feet. $2 \times 18 \times \frac{9}{\cos 35} \approx 395.5$ The area of the roof is approximately 395.5 square feet.
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 3 #9 Part B Rubric	
Score	**Description**
2	Student response includes each of the following 2 elements: • Decision of "no" with valid reason • Valid explanation Sample Student Response: A 10% decrease in the roof area obtained in Part A is about 356 square feet which means one rectangle would have an area of about 178 square feet. $0.9 \times 396 = 356.4$ $\frac{356.4}{2} = 178.2$ If the base of the rectangle remains 18 feet, then one side of the rectangle representing the roof will be 18 feet and the other side will be 178/18 which is about 9.9 feet or 10 feet. Then, I could form a right triangle to find the angle measure and use the equation $\cos(x) = \frac{9}{9.9}$ which gives an approximate value of 25°. The angle would be about $25° + 90° = 115°$, and this would not meet the minimum requirement of 117°. Notes: • The decision "no" can only be awarded 1 point if it is accompanied by a valid reason. • It is possible that a wrong surface area in Part A could yield a "yes" answer in Part B. Such answers should be considered correct if accompanied by a valid reason.
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 3 #9 Part C Rubric	
Score	**Description**
2	Student response includes each of the following 2 elements: • Greatest angle of approximately 122.7 • Valid work shown Sample Student Response: The builder's budget means that he can buy at most 11 bundles of shingles. $\frac{325}{27.75} \approx 11.71$ Because each bundle covers 35 square feet, the greatest surface area for one side of the roof is 192.5 square feet. This means that the greatest length the side of the roof can be is about 10.7 feet. $\frac{35 \times 11}{2} = 192.5$ $\frac{192.5}{18} \approx 10.69$ $\cos(z) = \frac{9}{10.69}$ $z \approx 32.6578°$ Therefore, the angle would need to be less than $32.6578° + 90°$ or less than $122.6578°$.
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Unit 3 #11 Part A Rubric	
Score	**Description**
2	Student response includes accurate instructions for steps 2-5. Sample student response: For step 2, draw any arc centered at point Z. Label the intersections of the arc and the angle point A and point B. For step 3, draw any arc with a radius greater than half of $\angle BZA$ centered at point A. For step 4, draw an arc centered at point B with the same radius as the arc used in step 3. Label the intersection of the arcs from step 3 and 4 point C. For step 5, draw a ray through point C with an end point at Z.
1	Student response includes accurate instructions for at least two of the steps based on descriptions to previous steps.
0	Student response is incorrect or irrelevant.

Unit 3 #11 Part B Rubric	
Score	**Description**
2	Student response includes a full explanation of each of the following 2 elements: • Accurate explanation that refers to the use of corresponding parts of congruent triangles and the definition of angle bisector • A logical sequence of statements that constitute a valid mathematical explanation or proof Sample Student Response: 1. $\overline{AZ} \cong \overline{BZ}$ [Both segments were drawn with the same compass setting, and all radii of a given circle are congruent.] 2. $\overline{AC} \cong \overline{BC}$ [Both segments were drawn with the same compass setting, and all radii of a given circle are congruent.] 3. $\overline{ZC} \cong \overline{ZC}$ [Reflexive Property of Congruence] 4. $\triangle AZC \cong \triangle BZC$ [Side-Side-Side Triangle Congruence] 5. $\angle AZC \cong \angle BZC$ [Corresponding parts/angles of congruent triangles are congruent.] 6. $\overline{ZC}$ bisects $\angle AZB$ [definition of angle bisector]
1	Student response includes a partial explanation.
0	Student response is incorrect or irrelevant.